Investigation of Kindergarten Students' Spatial Constructions

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This paper reports research into kindergarten students' spatial constructions and describes advancements in students' problem solving under adult guidance. In particular, it reports kindergarten students' attempts to match solid shapes with their respective nets and to interpret isometric drawings of stacked cubes.

Recent research investigating mathematical concepts and processes of children in their first year at school has been largely influenced by constructivist views of learning. For example, Fosnot (1990), Macmillan (1990) and Wright (1990) provide examples in the context of probability, money and number, respectively, of how kindergarten students can be guided towards a closer awareness of their own mathematical thinking through interactive communication.

Through social interaction in an instructional setting, students may be nurtured to proceed beyond their present state of development to a higher level of development, as emphasised in the writings of Vygotsky, who introduced the notion of zone of proximal development, defined as "the distance between the actual development level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). The zone of proximal development focuses on those intellectual capabilities which are in the process of maturation, though currently in an embryonic state - capabilities such that "what children can do with assistance today they will be able to do by themselves tomorrow" (Vygotsky, 1978, p. 87).

Piaget and Inhelder (1956) considered that children's representation of space is not a perceptual reading off of their spatial environment, but is constructed from prior active manipulation of that environment. Subscribing to this view, Wheatley and Cobb (1990) believe that children construct through their actions an image of an object that may later be re-presented and transformed.

As with the contribution of Piaget and Inhelder, the theory of Pierre and Dina van Hiele as it relates to children's levels of geometric thinking (van Hiele, 1959; van Hiele, 1986) has been very influential. The van Hieles postulated five levels of mental development in geometric thinking: recognition, analysis, ordering, deduction and rigour. At the first of these levels students identify and operate on shapes according to their appearance. At the second level students recognise and characterize shapes by their properties. Students can classify figures hierarchically, by ordering their properties, and give informal arguments to justify their classifications at the third level. Students can establish theorems within an axiomatic system at the fourth level and can reason formally about mathematical systems at the fifth level. It is claimed that most students at primary school level, the first seven years of formal schooling, do not progress beyond the second level. For students in transition, reliable classification is difficult (Fuys et al., 1988). Clements and Battista (1992) suggest a sixth level, termed 'pre-recognition', a level of thinking more primitive than, and probably pre-requisite to, van Hiele's Level 1. It is claimed that at this level students may be unable to identify common shapes because they lack the ability to form requisite visual images; that is, they may attend to only a subset of the shape's visual characteristics.

The theories of both Piaget and the van Hieles emphasize the role of students in actively constructing their own knowledge. Both stress the importance of challenging the learner's thinking; Piaget stresses the role of disequilibrium while van Hieles suggest 'crises of thinking'.

Piaget believed that the child's progressive organization of geometric ideas follows a definite order, and that this order is logical rather than historical in that initially topological relations, such as enclosure, connectedness and continuity, are constructed; later projective and Euclidean relations are developed (Gruber & Voneche, 1977). This has been termed the 'topological primacy thesis'. Research over the last two decades has tended not to support this theory (Clements & Battista, 1992).

Some confusion has resulted with respect to the use of the terms 'period', 'phase', 'stage' and 'level'. Von Glasersfeld and Kelley (1982) discussed the differences between them and noted that while the first three refer to stretches of time, 'level' does not refer to a stretch of time at all but implies a specific degree or height of some measurable feature or performance.

The research reported in this paper focuses upon two separate, yet related, studies involving kindergarten students' spatial constructions involving three dimensional shapes: a) matching solid shapes with their nets and b) interpreting isometric drawings of stacked cubes.

Matching solid shapes with their nets

Background to Study

While observing kindergarten students using Polydron interlocking squares and triangles to form three-dimensional shapes, the author noted that some students first formed the correct two-dimensional net before folding up the pieces to form the three-dimensional construct. This seemed to indicate that these students were able to mentally transform the three-dimensional shape into its corresponding two-dimensional net.

This prompted two questions:

a) Are kindergarten students able to associate foldout shapes (nets) with certain polyhedra?

b) What advances in the thinking of the students occur when they are challenged to justify their answers?

Bourgeois (1986) investigated with year three students a question similar to the first question.

Method

To facilitate investigation of these questions, a set of tasks was devised and presented to 33 students who were selected representatively from two kindergarten classes, the pupils being individually interviewed by the author. Each interview was recorded on video for subsequent analysis.

A cube, a triangular prism, a square pyramid and a tetrahedron were formed from Polydron interlocking squares and triangles. Also nets for each of these solids were formed from the Polydron material. The four nets (Figure 1) were laid out on the table in view of the interviewee, who was handed the cube and asked, as the interviewer pointed to the nets, "Which of these will fold up to make this shape?" When the interviewee had chosen a particular net, he/she was encouraged to explain why that had been chosen. Upon removal of the cube, a similar procedure was followed in turn for the triangular prism, square pyramid and tetrahedron.

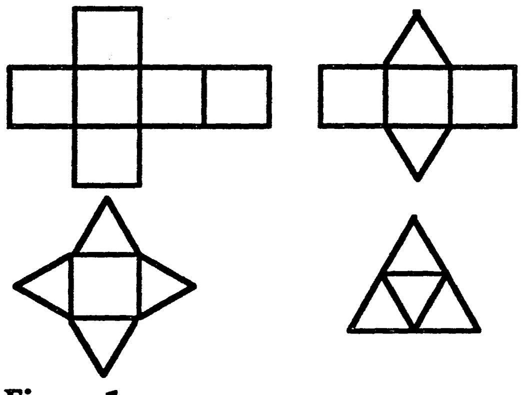


Figure 1

The interviewer asked additional questions which had the purpose of challenging the interviewee's thinking. If, for example, the interviewee responded that he/she had chosen a particular net "because the shapes (net and solid) are the same", the interviewer asked the child to explain in what way they were "the same". Particularly in those cases where an incorrect matching of net and solid had occurred, the interviewer asked the child to identify and count the shapes of the faces of the solid and compare these with the selected (incorrect) net.

Results

Table 1 provides details of the number of students who selected the correct net: a) initially or b) after interaction with the interviewer.

Table 1 Correct Responses for Matching Nets to Solids (n = 33)

Solid	Initially	After Interaction
Cube	30	2
Triangular Prism	7	14
Square Pyramid	17	5
Tetrahedron	24	5

A typical explanation given by students, who correctly matched the cube with its net, was "because it has all squares". Five students commented that the net would "fold up" to make the cube.

When attempting to match the triangular prism with its net, 13 of the students initially selected the net for the square pyramid, many of them explaining that "it has five shapes".

Discussion

Greatest difficulty was experienced by students when attempting to match the triangular prism and, to a lesser extent, the square pyramid with their respective nets. This outcome was expected by the researcher since both solids have five faces comprising a combination of squares and triangles; hence one could easily be confused with the other. That more children correctly matched the square pyramid with its net than matched the triangular prism with its net suggests that it is easier for students to imagine the folding up of the net to make the square pyramid. An alternative explanation is: prior to pupils' attempts to find the net for the square pyramid, they engaged in interactive communication with the interviewer as they attempted to find nets for each of the cube and triangular prism; this dialogue may have brought about advancements in some of the students' thinking, as suggested by the increasing number of correct responses for the square pyramid and for the tetrahedron. To test these possibilities, a further investigation was carried out with 10 more students for whom the order of presentation of solids was varied to cube, square pyramid, triangular prism and tetrahedron. The results of this further investigation, which are shown in Table 2, support the second hypothesis - that advancements in the student's thinking are attributable to interactive communication between the child and the interviewer.

Table 2 Correct Responses for Matching Nets to Solids (n = 10)

Solid	Initially	After Interaction
Cube	8	2
Square Pyramid	4	6
Triangular Prism	7	2
Tetrahedron	8	1

This advancement in thinking is illustrated in the following episode in which the interviewer (I) interviews Diana (D), aged 5 years 6 months:

I Which one would make this (holds up the triangular prism)?

D This one (pointing to the tetrahedron)

I Why?

D It has these shapes.

I What shape is this red shape (pointing to a square)?

D Square.

I Has this (net for tetrahedron) got any squares?

D No.

I So could this (net for tetrahedron) make it?

D No (Points to net for square pyramid).

I Oh, this one might make it? Let's have a think and see if it could. I How many squares has this got?

D One.

I What about that one (nodding towards the solid triangular prism)?

D Got one (shows one face).

I Is that all? No other squares?

D One there, one there (pointing to the other two square faces).

I Oh! How many squares does it have?

D Three!

I Has that got three squares (pointing to net for square pyramid)?

D No.

I (mimics) No! So what one is going to make this one?

D (Points to net for triangular prism)

I Why?

D It's got three squares.

I What else does it have?

D Two triangles.

I What about that one (pointing to solid triangular prism)?

D One, two. (She smiles and gestures with her arms in an attitude of obvious success.)

Following this Diana was presented with the square pyramid and asked to select its net. She reflected for a moment and then in a very confident manner pointed to the appropriate net. When asked for the reason, she responded without hesitation "because it's got one square and one, two, three, four of them (triangles)".

Interpreting isometric drawings of stacked cubes

Background to Study

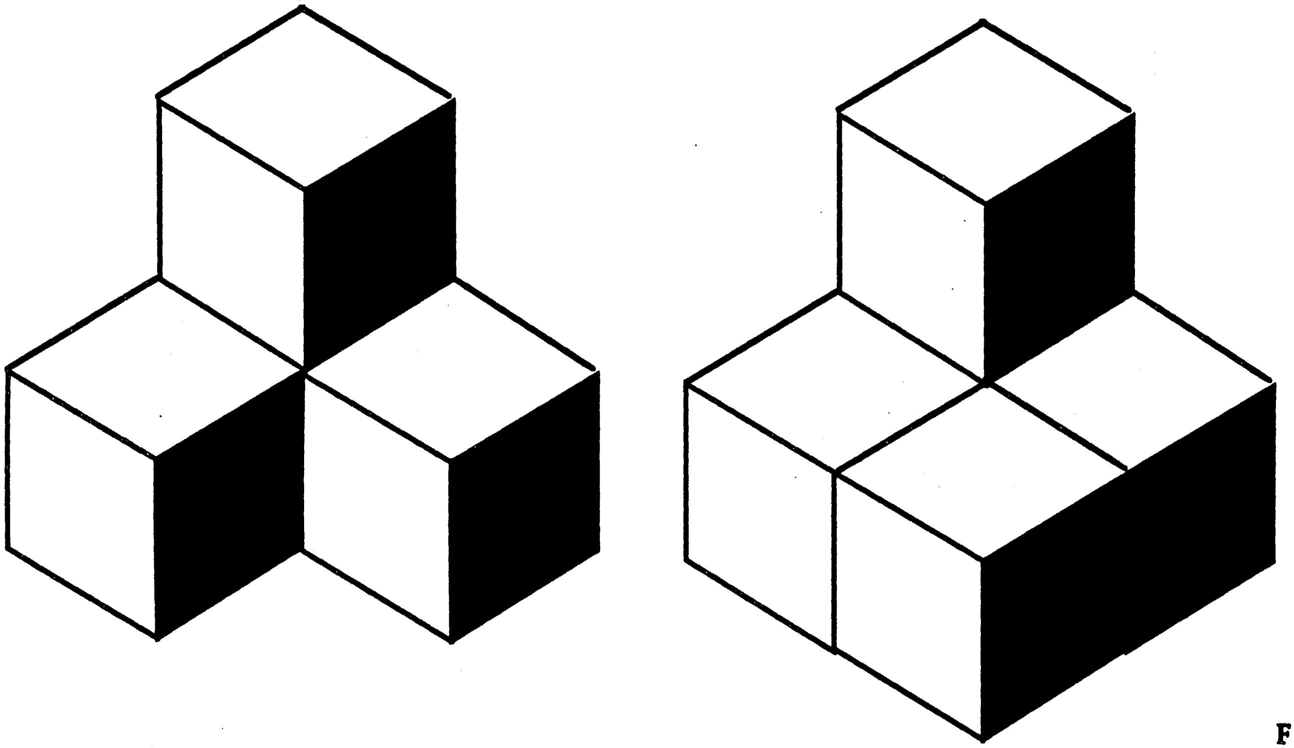
The researcher observed kindergarten students as they used coloured cubes to form structures as represented by two-dimensional drawings. These activities prompted the following questions:

a) Are kindergarten students able to correctly determine the number of cubes required to form a structure represented by a two-dimensional drawing where some cubes are hidden from view?

b) What advances in the thinking of the students occur as a result of interactive communication?

Method

To facilitate this investigation, 30 students were representatively selected from two kindergarten classes. In an individual interview, each pupil was presented with an isometric drawing of a stack of four cubes (Figure 2) and asked to state how many cubes he/she would need to form it. The same procedure was repeated for a stack of 5 cubes (Figure 3) and for a stack of 10 cubes (Figure 4). Where students appeared merely to count the visible cubes, the interviewer pointed to an elevated cube, asked whether it was on the table or "up in the air" and then asked how it could be "up in the air". When students gave a number of cubes greater than those which were visible, they were asked to state where the "hidden" ones were.



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igure 2: Stack of four cubes (left) and five cubes (right)

Results

Table 3 provides details of the number of students who stated the correct number of cubes: a) initially and b) following interaction with the interviewer.

Table 3 Correct Responses for Number of Cubes (n = 30)

Task	Initially	After Interaction
4-cube example	5	12
5-cube example	12	11
10-cube example	4	3

Those students who correctly stated that four cubes were required for the first example, when asked how they knew, responded that "there's another one underneath". A similar response was given for the second example.

Although only seven pupils were able to solve the third task, a further ten pupils realised that there were hidden cubes under three of the visible cubes, but either counted too many or too few.

Discussion

Increased initial success with the second task seems to result from the experience, including interactive communication, associated with the first task. This was also evident for the more difficult third task where only eight pupils were unaware of hidden cubes.

The following episode relates to the third task and involves Daniel (D), aged 5 years 5 months, and the interviewer (I). This illustrates students' mathematical reasoning as revealed in this study.

D Eight.

I How did you know eight?

D I counted them.

I Well, there must be some hiding ones.

D Yes, behind there.

I Whereabouts?

D Behind there.

I Which ones are they hiding under?

D That one (pointing to the highest) and ......(pause).. those two .....that means ten!

It seems reasonable to claim that asking Daniel to explain how he obtained his answer resulted in his reflecting upon his reasoning and in so doing he re-organised his understanding of this situation.

Conclusion

In both studies it was evident that advancements in spatial thinking of kindergarten students occurred following interactive communication with the interviewer. From the first, it can be concluded that a majority of kindergarten students can associate the cube, the triangular prism, the triangular pyramid and the tetrahedron with their respective nets. From the second, it may be concluded that a majority of kindergarten students are aware of "hidden" cubes in isometric drawings.

The results from both studies support Wright's (1994) contention that in the first year of school many children are under-challenged in mathematics.

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